

BIPOLAR INTERVAL VALUED INTUITIONISTIC FUZZY NECESSITY OPERATOR

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Abstract

In this paper we have introduced the necessity operator on a Bipolar Interval Valued Intuitionistic Fuzzy Subset of a Bipolar Interval Valued Intuitionistic Fuzzy Topological space and verified its property.

Keyword

Bipolar Interval Valued Intuitionistic Fuzzy Topological Space, Bipolar Interval Valued Intuitionistic Fuzzy Set.

1. Introduction:

Lee introduced the concept of Bipolar fuzzy set. In Bipolar Intuitionistic Fuzzy Topology the membership and non-membership degree of the fuzzy set lies in the range [0,1] and [-1,0][21]. In this paper we have introduced the Bipolar Interval Valued Intuitionistic Fuzzy necessity operator on a Bipolar Interval Valued Intuitionistic Fuzzy Subset of a Bipolar Interval Valued Intuitionistic Fuzzy Topological space and verified that the necessity operator on a Bipolar Interval Valued Intuitionistic Fuzzy Subset itself forms a Bipolar Interval Valued Intuitionistic Fuzzy Topological space.

2. Definition:

Let X be a non-empty set, and let A be a Bipolar interval valued intuitionistic fuzzy set on a Bipolar Interval Valued Intuitionistic Topological Space BIVIFTS(X), then the necessity operator on A is defined as

$$\text{i. } []A = \left\{ /x, \left[\begin{array}{l} \underline{\underline{\theta}}^P(x), \underline{\underline{\theta}}^P(x) \\ \underline{\underline{\theta}}_{AL}^{AU}(x), \underline{\underline{\theta}}_{AU}^{AL}(x) \end{array} \right], \left[\begin{array}{l} 1 \square \underline{\underline{\theta}}^P(x), 1 \square \underline{\underline{\theta}}^P(x) \\ \square 1 + \underline{\underline{\theta}}_{AU}^{AU}(x), \square 1 + \underline{\underline{\theta}}_{AL}^{AL}(x) \end{array} \right], \right| x \square x \right\}$$

2.1. Theorem:

Let (X, \mathbb{I}) be a Bipolar Interval Valued Intuitionistic Fuzzy Topological Space (BIVIFTS). Based on the necessity operator on a Bipolar Interval Valued Intuitionistic Fuzzy set A on X, we can also construct several BIVIFTSs on X as

$$\mathbb{B}_N = \left\{ []A \mid A \sqsubseteq \mathbb{B} \right\}$$

i.e., the necessity operator defined in the above definition itself forms a topology.

Proof:

In order to prove the topology we have to prove the following

Let S be a set and \mathcal{A} be a family of bipolar interval valued intuitionistic fuzzy subset of S. The family is called a Bipolar Interval Valued Intuitionistic Fuzzy Topology (BIVIFT) on S if satisfies the following axioms

i. $0_s, 1_s \sqsubseteq \mathbb{B}$

ii. If $\{A_i; i \in I\} \sqsubseteq \mathbb{B}$, then $\bigcap_{i=1}^{\square} A_i \sqsubseteq \mathbb{B}$

iii. If $A_1, A_2, A_3 \dots A_n \sqsubseteq \mathbb{B}$, then $\bigcup_{i=1}^n A_i \sqsubseteq \mathbb{B}$

Let A_1, A_2, \dots, A_n be Bipolar interval valued intuitionistic fuzzy subsets on a Bipolar Interval Valued Intuitionistic Topological Space BIVIFTS(X).

To prove necessity operator is a Bipolar Interval Valued Intuitionistic Topological Space BIVIFTS(X)

i. obviously $0_s, 1_s \sqsubseteq \mathbb{B}_N$

ii.

$$A \sqsubseteq B = \left(\left[\mathbb{B}_{(A \sqsubseteq B)_L}^P(x), \mathbb{B}_{(A \sqsubseteq B)_U}^P(x) \right], \left[\mathbb{B}_{(A \sqsubseteq B)_L}^N(x), \mathbb{B}_{(A \sqsubseteq B)_U}^N(x) \right] \right) \mid x \in X$$

where

$$\mathbb{B}_{(A \sqsubseteq B)_L}^P(x) = \min \left\{ \mathbb{B}_{A_L}^P(x), \mathbb{B}_{B_U}^P(x) \right\}$$

$$\begin{aligned} \max_{(A \square B)U}^p(x) &= \max \left\{ \max_{AU}^p(x), \max_{BU}^p(x) \right\} \\ \max_{(A \square B)L}^N(x) &= \max \left\{ \max_{AL}^N(x), \max_{BL}^N(x) \right\} \end{aligned}$$

$$\mathbb{E}_{(A \square B)_U}^N(x) = \min \left\{ \mathbb{E}_{AU}^N(x), \mathbb{E}_{BU}^N(x) \right\}$$

$$\mathbb{I}_{(A \square B)_L}^p(x) = \min \left\{ \mathbb{I}_{AL}^p(x), \mathbb{I}_{BL}^p(x) \right\}$$

$$\mathbb{I}_{(A \square B)_U}^p(x) = \max \left\{ \mathbb{I}_{AU}^p(x), \mathbb{I}_{BU}^p(x) \right\}$$

$$\max_{\{t \in \mathbb{R}\}} \left(x \right) = \max \left\{ \left| x \right|, \left| -x \right| \right\}$$

$$\mathbb{E}^N_{(\epsilon \square p)_U}(x) = \min \left\{ \mathbb{E}^N_{\mu_U}(x), \mathbb{E}^N_{p_U}(x) \right\}$$

$$\square(A \Box B)U(x) = \min\{f_AU(x), f_BU(x)\}$$

$$= \left[x, \left(p_{A_1} \right)_{A_2} \right]_L (x, \left(p_{A_1} \right)_{A_2})_U,$$

$$\square [] A \square [] A = \begin{cases} \square \square \left\langle \begin{array}{l} \left[x, \square \square \square [] A_1 \square [] A_2 \right] L () x, \square \square \square [] A_1 \square [] A_2 \right] U (x) \end{array} \right\rangle, \\ \square \square \left\langle \begin{array}{l} \square \square \square [] A_1 \square [] A_2 \right] L (x), \square \square \square [] A_1 \square [] A_2 \right] U (x) \end{array} \right\rangle, \\ \square \square \left\langle \begin{array}{l} \square \square \square [] A_1 \square [] A_2 \right] L (x), \square \square \square [] A_1 \square [] A_2 \right] U (x) \end{array} \right\rangle, \\ \square \square \left\langle \begin{array}{l} \square \square \square [] A_1 \square [] A_2 \right] L (x), \square \square \square [] A_1 \square [] A_2 \right] U (x) \end{array} \right\rangle, \end{cases} | x \square X$$

where

$$\min\left\{\left[\begin{array}{c} p \\ \square_{A_1} \square_{A_2} L \end{array}\right]_{A_1 L}, \left[\begin{array}{c} p \\ \square_{A_2} L \end{array}\right]_{A_2 L}\right\}$$

$$\max_{\left[\begin{smallmatrix} p \\ A_1 \square A_2 \end{smallmatrix} \right] U} \left(x \right) = \max \left\{ \max_{\left[\begin{smallmatrix} p \\ A_1 U \end{smallmatrix} \right]} \left(x \right), \max_{\left[\begin{smallmatrix} p \\ A_2 U \end{smallmatrix} \right]} \left(x \right) \right\}$$

$$\mathbb{E}^N_{\left(\begin{array}{c} \\ A_1 \end{array}\right) \square \left(\begin{array}{c} \\ A_2 \end{array}\right)_L}(x) = \max \left\{ \mathbb{E}^N_{\left(\begin{array}{c} \\ A_1 L \end{array}\right)}(x), \mathbb{E}^N_{\left(\begin{array}{c} \\ A_2 L \end{array}\right)}(x) \right\}$$

$$\min\left\{\mathbb{E}_{[A_1U]}^N(x), \mathbb{E}_{[A_2U]}^N(x)\right\}$$

$$\mathbb{I}_{\left(\begin{array}{c} p \\ A_1 \square A_2 \end{array}\right)_L}(x) = \min \left\{ \mathbb{I}_{A_1 L}^p(x), \mathbb{I}_{A_2 L}^p(x) \right\}$$

$$\max_{\left[\begin{smallmatrix} p \\ A_1 \square A_2 \end{smallmatrix} \right] U} \left(x \right) = \max \left\{ \left[\begin{smallmatrix} p \\ A_1 U \end{smallmatrix} \right] \left(x \right), \left[\begin{smallmatrix} p \\ A_2 U \end{smallmatrix} \right] \left(x \right) \right\}$$

$$\max_{\{x\}} \left(\max_{[A_1]L} x \right) = \max_{[A_1]L} \left(\max_{[A_2]L} x \right)$$

$$\mathbb{I}_{\left(\begin{array}{c} N \\ A_1 \sqcup A_2 \end{array}\right)U}(x) = \min \left\{ \mathbb{I}_{\begin{array}{c} N \\ A_1 U \end{array}}(x), \mathbb{I}_{\begin{array}{c} N \\ A_2 U \end{array}}(x) \right\}$$

then

$$\mathbb{E}^p_{\left(\begin{bmatrix} \cdot \\ \cdot \end{bmatrix}_{A_1}, \begin{bmatrix} \cdot \\ \cdot \end{bmatrix}_{A_2}\right)_L}(x) = \min \left\{ \mathbb{E}^p_{A_1 L}(x), \mathbb{E}^p_{A_2 L}(x) \right\}$$

$$\mathbb{E}^p_{([])_{A_1} \square ([])_{A_2}}(x) = \max \left\{ \mathbb{E}^p_{A_1 U}(x), \mathbb{E}^p_{A_2 U}(x) \right\}$$

$$\mathbb{E}^N([A_1 \sqcup A_2]_L)(x) = \max \left\{ \mathbb{E}^N_{A_1 L}(x), \mathbb{E}^N_{A_2 L}(x) \right\}$$

$$\mathbb{E}_{(\{A_1 \square [A_2]\}_U)}^N(x) = \min\left\{\mathbb{E}_{A_1 U}^N(x), \mathbb{E}_{A_2 U}^N(x)\right\}$$

$$1 \square \mathbb{E}_{(\{A_1 \square [A_2]\}_L)}^p(x) = \min\left\{1 \square \mathbb{E}_{A_1 L}^p(x), 1 \square \mathbb{E}_{A_2 L}^p(x)\right\}$$

$$1 \square \mathbb{E}_{(\{A_1 \square [A_2]\}_U)}^p(x) = \max\left\{1 \square \mathbb{E}_{A_1 U}^p(x), 1 \square \mathbb{E}_{A_2 U}^p(x)\right\}$$

$$1 \square \mathbb{E}_{(\{A_1 \square [A_2]\}_L)}^N(x) = \max\left\{1 \square \mathbb{E}_{A_1 L}^N(x), 1 \square \mathbb{E}_{A_2 L}^N(x)\right\}$$

$$1 \square \mathbb{E}_{(\{A_1 \square [A_2]\}_U)}^N(x) = \min\left\{1 \square \mathbb{E}_{A_1 U}^N(x), 1 \square \mathbb{E}_{A_2 U}^N(x)\right\}$$

$$\begin{aligned} \square_{[A_1 \square [A_2]} &= \left\{ \begin{array}{l} \left[x, \mathbb{E}_{(\{A_1 \square [A_2]\}_L)}^p(x), \mathbb{E}_{(\{A_1 \square [A_2]\}_U)}^N(x) \right], \\ \left[\mathbb{E}_{(\{A_1 \square [A_2]\}_L)}^p(x), \mathbb{E}_{(\{A_1 \square [A_2]\}_U)}^N(x) \right], \\ \left[(\{A_1 \square [A_2]\}_L), (\{A_1 \square [A_2]\}_U) \right] \end{array} \right| x \square X \end{aligned}$$

$$\begin{aligned} \square_{[A_1 \square [A_2 \square \dots [A_i]]} &= \left\{ \begin{array}{l} \left[x, \mathbb{E}_{(\{A_1 \square [A_2 \square \dots [A_i]\}_L)}^p(x), \mathbb{E}_{(\{A_1 \square [A_2 \square \dots [A_i]\}_U)}^N(x) \right], \\ \left[\mathbb{E}_{(\{A_1 \square [A_2 \square \dots [A_i]\}_L)}^p(x), \mathbb{E}_{(\{A_1 \square [A_2 \square \dots [A_i]\}_U)}^N(x) \right], \\ \left[\mathbb{E}_{(\{A_1 \square [A_2 \square \dots [A_i]\}_L)}^p(x), \mathbb{E}_{(\{A_1 \square [A_2 \square \dots [A_i]\}_U)}^N(x) \right], \\ \left[\mathbb{E}_{(\{A_1 \square [A_2 \square \dots [A_i]\}_L)}^p(x), \mathbb{E}_{(\{A_1 \square [A_2 \square \dots [A_i]\}_U)}^N(x) \right] \end{array} \right| x \square X \end{aligned}$$

where

$$\mathbb{E}_{(\{A_1 \square [A_2 \square \dots [A_i]\}_L)}^p(x) = \min\left\{\mathbb{E}_{A_1 L}^p(x), \mathbb{E}_{A_2 L}^p(x), \dots, \mathbb{E}_{A_i L}^p(x)\right\}$$

$$\mathbb{E}_{(\{A_1 \square [A_2 \square \dots [A_i]\}_U)}^p(x) = \max\left\{\mathbb{E}_{A_1 U}^p(x), \mathbb{E}_{A_2 U}^p(x), \dots, \mathbb{E}_{A_i U}^p(x)\right\}$$

$$\mathbb{E}_{(\{A_1 \square [A_2 \square \dots [A_i]\}_L)}^N(x) = \max\left\{\mathbb{E}_{A_1 L}^N(x), \mathbb{E}_{A_2 L}^N(x), \dots, \mathbb{E}_{A_i L}^N(x)\right\}$$

$$\mathbb{E}_{(\{A_1 \square [A_2 \square \dots [A_i]\}_U)}^N(x) = \min\left\{\mathbb{E}_{A_1 U}^N(x), \mathbb{E}_{A_2 U}^N(x), \dots, \mathbb{E}_{A_i U}^N(x)\right\}$$

$$\mathbb{E}_{(\{A_1 \square [A_2 \square \dots [A_i]\}_L)}^p(x) = \min\left\{\mathbb{E}_{A_1 L}^p(x), \mathbb{E}_{A_2 L}^p(x), \dots, \mathbb{E}_{A_i L}^p(x)\right\}$$

$$\mathbb{E}_{(\{A_1 \square [A_2 \square \dots [A_i]\}_U)}^p(x) = \max\left\{\mathbb{E}_{A_1 U}^p(x), \mathbb{E}_{A_2 U}^p(x), \dots, \mathbb{E}_{A_i U}^p(x)\right\}$$

$$\mathbb{E}_{(\{A_1 \square [A_2 \square \dots [A_i]\}_L)}^N(x) = \max\left\{\mathbb{E}_{A_1 L}^N(x), \mathbb{E}_{A_2 L}^N(x), \dots, \mathbb{E}_{A_i L}^N(x)\right\}$$

$$\mathbb{E}_{(\{A_1 \square [A_2 \square \dots [A_i]\}_U)}^N(x) = \min\left\{\mathbb{E}_{A_1 U}^N(x), \mathbb{E}_{A_2 U}^N(x), \dots, \mathbb{E}_{A_i U}^N(x)\right\}$$

then

$$\mathbb{E}_{(\{A_1 \square [A_2 \square \dots [A_i]\}_L)}^p(x) = \min\left\{\mathbb{E}_{A_1 L}^p(x), \mathbb{E}_{A_2 L}^p(x), \dots, \mathbb{E}_{A_i L}^p(x)\right\}$$

$$\begin{aligned}
 \exists^P_{([A_1 \square [A_2 \square \dots [A_i]_U])} (x) &= \max \left\{ \exists^P_{A_1 U}(x), \exists^P_{A_2 U}(x), \dots, \exists^P_{A_i U}(x) \right\} \\
 \exists^N_{([A_1 \square [A_2 \square \dots [A_i]_L])} (x) &= \max \left\{ \exists^N_{A_1 L}(x), \exists^N_{A_2 L}(x), \dots, \exists^N_{A_i L}(x) \right\} \\
 \exists^N_{([A_1 \square [A_2 \square \dots [A_i]_U])} (x) &= \min \left\{ \exists^N_{A_1 U}(x), \exists^N_{A_2 U}(x), \dots, \exists^N_{A_i U}(x) \right\} \\
 1 \exists^P_{([A_1 \square [A_2 \square \dots [A_i]_L])} (x) &= \min \left\{ 1 \exists^P_{A_1 L}(x), 1 \exists^P_{A_2 L}(x), \dots, 1 \exists^P_{A_i L}(x) \right\} \\
 1 \exists^P_{([A_1 \square [A_2 \square \dots [A_i]_U])} (x) &= \max \left\{ 1 \exists^P_{A_1 U}(x), 1 \exists^P_{A_2 U}(x), \dots, 1 \exists^P_{A_i U}(x) \right\} \\
 1 \exists^N_{([A_1 \square [A_2 \square \dots [A_i]_L])} (x) &= \max \left\{ 1 \exists^N_{A_1 L}(x), 1 \exists^N_{A_2 L}(x), \dots, 1 \exists^N_{A_i L}(x) \right\} \\
 1 \exists^N_{([A_1 \square [A_2 \square \dots [A_i]_U])} (x) &= \min \left\{ 1 \exists^N_{A_1 U}(x), 1 \exists^N_{A_2 U}(x), \dots, 1 \exists^N_{A_i U}(x) \right\} \\
 \square [A_1 \square [A_2 \square \dots [A_i]_A] &= \left\{ \begin{array}{l} \exists^P_{A_1 L}(x), \exists^P_{A_2 L}(x), \dots, \exists^P_{A_i L}(x), \\ \exists^N_{A_1 U}(x), \exists^N_{A_2 U}(x), \dots, \exists^N_{A_i U}(x), \\ 1 \exists^P_{A_1 L}(x), 1 \exists^P_{A_2 L}(x), \dots, 1 \exists^P_{A_i L}(x), \\ 1 \exists^N_{A_1 U}(x), 1 \exists^N_{A_2 U}(x), \dots, 1 \exists^N_{A_i U}(x) \end{array} \right\} | x \square X
 \end{aligned}$$

Hence the arbitrary union of Bipolar Interval Valued Intuitionistic Necessity Operators is in Bipolar Interval Valued Intuitionistic Fuzzy Topology τ .

iii.

$$A \square B = \left\langle \begin{array}{l} \left[x, \exists^P_{(A \square B)_L}(x), \exists^P_{(A \square B)_U}(x) \right], \left[\exists^N_{(A \square B)_L}(x), \exists^N_{(A \square B)_U}(x) \right], \\ \left[\exists^P_{(A \square B)_L}(\not x), \exists^P_{(A \square B)_U}(\not x) \right], \left[\exists^N_{(A \square B)_L}(\not x), \exists^N_{(A \square B)_U}(\not x) \right] \end{array} \right\rangle | x \square X$$

where

$$\exists^P_{(A \square B)_L}(x) = \max \left\{ \exists^P_{AL}(x), \exists^P_{BL}(x) \right\}$$

$$\exists^P_{(A \square B)_U}(x) = \min \left\{ \exists^P_{AU}(x), \exists^P_{BU}(x) \right\}$$

$$\exists^N_{(A \square B)_L}(x) = \min \left\{ \exists^N_{AL}(x), \exists^N_{BL}(x) \right\}$$

$$\exists^N_{(A \square B)_U}(x) = \max \left\{ \exists^N_{AU}(x), \exists^N_{BU}(x) \right\}$$

$$\exists^P_{(A \square B)_L}(x) = \max \left\{ \exists^P_{AL}(x), \exists^P_{BL}(x) \right\}$$

$$\exists^P_{(A \square B)_U}(x) = \min \left\{ \exists^P_{AU}(x), \exists^P_{BU}(x) \right\}$$

$$\exists^N_{(A \square B)_L}(x) = \min \left\{ \exists^N_{AL}(x), \exists^N_{BL}(x) \right\}$$

$$\mathbb{I}_{(A \square B)_U}^N(x) = \max \left\{ \mathbb{I}_{AU}^N(x), \mathbb{I}_{BU}^N(x) \right\}$$

then

$$([]_A \square []_A) = \begin{cases} \left[\mathbb{I}_{A_1 \square []_{A_2} L}^p(x), \mathbb{I}_{A_1 \square []_{A_2} U}^p(x) \right], \\ \left[\mathbb{I}_{[]_{A_1 \square []_{A_2}} L}^p(x), \mathbb{I}_{[]_{A_1 \square []_{A_2}} U}^p(x) \right], \\ \left[\mathbb{I}_{[]_{A_1 \square []_{A_2}} L}^p(x), \mathbb{I}_{[]_{A_1 \square []_{A_2}} U}^p(x) \right], \\ \left[\mathbb{I}_{[]_{A_1 \square []_{A_2}} L}^p(x), \mathbb{I}_{[]_{A_1 \square []_{A_2}} U}^p(x) \right], \end{cases} | x \in X$$

where

$$\begin{aligned} \mathbb{I}_{[]_{A_1 \square []_{A_2}} L}^p(x) &= \max \left\{ \mathbb{I}_{A_1 L}^p(x), \mathbb{I}_{A_2 L}^p(x) \right\} \\ \mathbb{I}_{[]_{A_1 \square []_{A_2}} U}^p(x) &= \min \left\{ \mathbb{I}_{A_1 U}^p(x), \mathbb{I}_{A_2 U}^p(x) \right\} \\ \mathbb{I}_{[]_{A_1 \square []_{A_2}} L}^N(x) &= \min \left\{ \mathbb{I}_{A_1 L}^N(x), \mathbb{I}_{A_2 L}^N(x) \right\} \\ \mathbb{I}_{[]_{A_1 \square []_{A_2}} U}^N(x) &= \max \left\{ \mathbb{I}_{A_1 U}^N(x), \mathbb{I}_{A_2 U}^N(x) \right\} \\ \mathbb{I}_{[]_{A_1 \square []_{A_2}} L}^p(x) &= \max \left\{ \mathbb{I}_{A_1 L}^p(x), \mathbb{I}_{A_2 L}^p(x) \right\} \\ \mathbb{I}_{[]_{A_1 \square []_{A_2}} U}^p(x) &= \min \left\{ \mathbb{I}_{A_1 U}^p(x), \mathbb{I}_{A_2 U}^p(x) \right\} \\ \mathbb{I}_{[]_{A_1 \square []_{A_2}} L}^N(x) &= \min \left\{ \mathbb{I}_{A_1 L}^N(x), \mathbb{I}_{A_2 L}^N(x) \right\} \\ \mathbb{I}_{[]_{A_1 \square []_{A_2}} U}^N(x) &= \max \left\{ \mathbb{I}_{A_1 U}^N(x), \mathbb{I}_{A_2 U}^N(x) \right\} \end{aligned}$$

then

$$\begin{aligned} \mathbb{I}_{[]_{A_1 \square []_{A_2}} L}^p(x) &= \max \left\{ \mathbb{I}_{A_1 L}^p(x), \mathbb{I}_{A_2 L}^p(x) \right\} \\ \mathbb{I}_{[]_{A_1 \square []_{A_2}} U}^p(x) &= \min \left\{ \mathbb{I}_{A_1 U}^p(x), \mathbb{I}_{A_2 U}^p(x) \right\} \\ \mathbb{I}_{[]_{A_1 \square []_{A_2}} L}^N(x) &= \min \left\{ \mathbb{I}_{A_1 L}^N(x), \mathbb{I}_{A_2 L}^N(x) \right\} \\ \mathbb{I}_{[]_{A_1 \square []_{A_2}} U}^N(x) &= \max \left\{ \mathbb{I}_{A_1 U}^N(x), \mathbb{I}_{A_2 U}^N(x) \right\} \\ 1 \square \mathbb{I}_{[]_{A_1 \square []_{A_2}} L}^p(x) &= \max \left\{ 1 \square \mathbb{I}_{A_1 L}^p(x), 1 \square \mathbb{I}_{A_2 L}^p(x) \right\} \\ 1 \square \mathbb{I}_{[]_{A_1 \square []_{A_2}} U}^p(x) &= \min \left\{ 1 \square \mathbb{I}_{A_1 U}^p(x), 1 \square \mathbb{I}_{A_2 U}^p(x) \right\} \\ 1 \square \mathbb{I}_{[]_{A_1 \square []_{A_2}} L}^N(x) &= \min \left\{ 1 \square \mathbb{I}_{A_1 L}^N(x), 1 \square \mathbb{I}_{A_2 L}^N(x) \right\} \end{aligned}$$

$$\begin{aligned}
 & 1 \square \bar{\square}^N_{([A_1 \square [A_2]_U]}(x) = \max \left\{ 1 \square \bar{\square}_{A_1 U}^N(x), 1 \square \bar{\square}_{A_2 U}^N(x) \right\} \\
 & \square [] A_1 \square [] A_2 = \left\{ \begin{array}{l} x, \bar{\square}^p_{([A_1 \square [A_2]_L]}(x), \bar{\square}^p_{([A_1 \square [A_2]_U]}(x), \\ \bar{\square}^N_{([A_1 \square [A_2]_L]}(x), \bar{\square}^N_{([A_1 \square [A_2]_U]}(x), \\ 1 \square \bar{\square}^p_{([A_1 \square [A_2]_L]}(x), 1 \square \bar{\square}^p_{([A_1 \square [A_2]_U]}(x), \\ 1 \square \bar{\square}^N_{([A_1 \square [A_2]_L]}(x), 1 \square \bar{\square}^N_{([A_1 \square [A_2]_U]}(x) \end{array} \right\} | x \square X \bar{\square}^N \\
 & \square [] A_1 \square [] A_2 \square \dots [] A_i = \left\{ \begin{array}{l} x, \bar{\square}^p_{([A_1 \square [A_2 \square \dots [A_i]_L]}(x), \bar{\square}^p_{([A_1 \square [A_2 \square \dots [A_i]_U]}(x), \\ \bar{\square}^N_{([A_1 \square [A_2 \square \dots [A_i]_L]}(x), \bar{\square}^N_{([A_1 \square [A_2 \square \dots [A_i]_U]}(x), \\ \bar{\square}^p_{([A_1 \square [A_2 \square \dots [A_i]_L]}(x), \bar{\square}^p_{([A_1 \square [A_2 \square \dots [A_i]_U]}(x), \\ \bar{\square}^N_{([A_1 \square [A_2 \square \dots [A_i]_L]}(x), \bar{\square}^N_{([A_1 \square [A_2 \square \dots [A_i]_U]}(x) \end{array} \right\} | x \square X \bar{\square}
 \end{aligned}$$

where

$$\bar{\square}^p_{([A_1 \square [A_2 \square \dots [A_i]_L]}(x) = \max \left\{ \bar{\square}^p_{[A_1 L}(x), \bar{\square}^p_{[A_2 L}(x), \dots, \bar{\square}^p_{[A_i L}(x) \right\}$$

$$\bar{\square}^p_{([A_1 \square [A_2 \square \dots [A_i]_U]}(x) = \min \left\{ \bar{\square}^p_{[A_1 U}(x), \bar{\square}^p_{[A_2 U}(x), \dots, \bar{\square}^p_{[A_i U}(x) \right\}$$

$$\bar{\square}^N_{([A_1 \square [A_2 \square \dots [A_i]_L]}(x) = \min \left\{ \bar{\square}^N_{[A_1 L}(x), \bar{\square}^N_{[A_2 L}(x), \dots, \bar{\square}^N_{[A_i L}(x) \right\}$$

$$\bar{\square}^N_{([A_1 \square [A_2 \square \dots [A_i]_U]}(x) = \max \left\{ \bar{\square}^N_{[A_1 U}(x), \bar{\square}^N_{[A_2 U}(x), \dots, \bar{\square}^N_{[A_i U}(x) \right\}$$

$$\bar{\square}^p_{([A_1 \square [A_2 \square \dots [A_i]_L]}(x) = \max \left\{ \bar{\square}^p_{[A_1 L}(x), \bar{\square}^p_{[A_2 L}(x), \dots, \bar{\square}^p_{[A_i L}(x) \right\}$$

$$\bar{\square}^p_{([A_1 \square [A_2 \square \dots [A_i]_U]}(x) = \min \left\{ \bar{\square}^p_{[A_1 U}(x), \bar{\square}^p_{[A_2 U}(x), \dots, \bar{\square}^p_{[A_i U}(x) \right\}$$

$$\bar{\square}^N_{([A_1 \square [A_2 \square \dots [A_i]_L]}(x) = \min \left\{ \bar{\square}^N_{[A_1 L}(x), \bar{\square}^N_{[A_2 L}(x), \dots, \bar{\square}^N_{[A_i L}(x) \right\}$$

$$\bar{\square}^N_{([A_1 \square [A_2 \square \dots [A_i]_U]}(x) = \max \left\{ \bar{\square}^N_{[A_1 U}(x), \bar{\square}^N_{[A_2 U}(x), \dots, \bar{\square}^N_{[A_i U}(x) \right\}$$

then

$$\bar{\square}^P_{([A_1 \square [A_2 \square \dots [A_i]_L]}(x) = \max \left\{ \bar{\square}^P_{[A_1 L}(x), \bar{\square}^P_{[A_2 L}(x), \dots, \bar{\square}^P_{[A_i L}(x) \right\}$$

Hence the finite intersection of Bipolar Interval Valued Intuitionistic Necessity Operators is in Bipolar Interval Valued Intuitionistic Fuzzy Topology τ .

Hence the Bipolar Interval Valued Intuitionistic Necessity Operators itself forms a Bipolar Interval Valued Intuitionistic Fuzzy Topology τ .

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